

Unit 15
Day 2
The Remainder Theorem

Remainder Theorem: If the polynomial $P(x)$ is divided by $x-k$, the remainder is $P(k)$

1) $P(x) = x^3 + 8x^2 - 5x - 84$ $k = -5$

$$\begin{array}{r|rrrr} -5 & 1 & 8 & -5 & -84 \\ & & -5 & -15 & 100 \\ \hline & 1 & 3 & -20 & 16 \end{array}$$

remainder $\therefore f(-5) = 16$

$$\begin{aligned} f(-5) &= (-5)^3 + 8(-5)^2 - 5(-5) - 84 \\ &= -125 + 200 + 25 - 84 = 16 \end{aligned}$$

2) $f(x) = 2x^3 + 3x^2 - 8x - 12$ $k = -2$

Find $F(k)$

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -8 & -12 \\ & & -4 & 2 & 12 \\ \hline & 2 & -1 & -6 & 0 \end{array}$$

$f(-2) = 0$

$$\begin{aligned} f(-2) &= 2(-2)^3 + 3(-2)^2 - 8(-2) - 12 \\ &= -16 + 12 + 16 - 12 = 0 \end{aligned}$$

When is the remainder theorem useful?

Is $P(x) = -2x^{19} + 8x^{17} - 6x^{10} + x^8 + 12$ divisible by $x-2$?

$$\begin{array}{r|rrrrrrrrrrrrrrrrrrrr} 2 & -2 & 0 & 8 & 0 & 0 & \dots & -6 & 0 & 1 & 0 & 0 & \dots & 12 \\ & & -4 & -8 & 0 & 0 & & 0 & -12 & -24 & -46 \\ \hline & -2 & -4 & 0 & 0 & 0 & 0 & 0 & -12 & -23 & -46 \end{array}$$

$P(2) = -5876$

Is $P(x) = x^{26} - 6x^{18} + 3$ divisible by $x-i$?

$$\begin{aligned} P(i) &= (i)^{26} - 6(i)^{18} + 3 \\ &= i^2 - 6i^2 + 3 \\ &= -1 + 6 + 3 = 8 \end{aligned}$$

$P(i) = 8$
No

Extra problems:

1) Is $P(x) = x^{99} - 2x^{52} + x^2$ divisible by $x+1$?

2) Is $P(x) = x^{101} + 3x^{20} + x^3$ divisible by $x-i$?

3) Find the value of k so that $(x^2 + 4x + 8) \div (x - k)$ has a remainder of 4.

$$k^2 + 4k + 8 = 4 \quad F(k) = 4$$
$$x^2 + 4x + 8 = 4$$

HW pg 290-291 1-4 all, 20-30 even & Extra problems